Functions can be seen as input-output relationships. The independent variable in a function is the input value (the value on which the dependent variable depends). It is usually the value that you are given. The dependent variable in a function is the output value, which may change depending on the input.

Two Variables

One of the simplest types of functional relationship has only two terms. It has the form $y = cx$, where $c$ is a number and $y$ and $x$ are the variables.

Examples: $y = 5x$, $y = \frac{1}{3}x$, $y = -2x$, $y = x$

Generally, these functions are written in such a way that the variable that is multiplied by the number $c$ is the independent variable. This is not always the case, however. You need to consider the meanings of the variables to determine which is independent and which is dependent.

EXAMPLE 1

Wilfred is paid $20 for each car he washes. If the number of cars he washes is $c$ and the total amount he earns is $p$, then the relationship between them can be expressed as $p = 20c$. What are the independent and dependent quantities in this relationship?

Wilfred is paid $20 for each car that he washes. So the total amount he gets paid depends on the number of cars he washes. Thus, his payment, $p$, is the dependent variable. The independent variable is the one that affects the dependent variable. Since the number of cars is given in order to determine the amount that Wilfred gets paid, the independent variable is $c$, the number of cars he washes.

Quick Check 1

1a. Henry can wash 4 windows per hour at his job. If $w$ is the number of windows he washes and $h$ is the number of hours he works, then $w = 4h$. Which is the independent variable in this relationship?

1b. Each week, Amanda plants 2 flowers in her garden. If $w$ is the number of weeks that she plants flowers, and $f$ is the total number of flowers she plants, which would be the dependent variable in a function relating them?
More Than Two Variables

Functional relationships can be more complicated when they involve more than two variables. There will always be one dependent variable, which depends on all the other variables. The other variables will be independent. The values of the independent variables are usually given.

EXAMPLE 2

The price of a moving truck is $40 plus $5 per hour and $0.10 per mile driven. The relationship between the number of hours for which the truck is rented, $h$, the number of miles driven, $m$, and the total price, $p$, is given by the equation $p = 40 + 5h + 0.10m$. Identify the independent and dependent variables in this relationship. This equation contains three variables. The variables are $p$, the total price; $m$, the number of miles driven; and $h$, the number of hours rented. The total price depends on the number of hours rented, $h$, and the number of miles, $m$. So $p$ is the dependent variable. Because the values of $h$ and $m$ are given in order to find the value of $p$, $h$ and $m$ are the independent variables.

Quick Check 2

2a. Al spent $100 on a lawnmower. He charges $10 per lawn to mow people’s lawns. He is also paid $5 per week by his mother. If $P$ is Al’s total profit, $w$ is the number of weeks he works, and $x$ is the number of lawns mowed, they can be related with the equation $P = 10x - 100 + 5w$. Identify the independent variable(s).

2b. James has a coupon card for his local movie theater. For every 10 movies he buys tickets for, he receives 1 pass for a free movie. Also, for every $50 he spends on concessions, he receives 1 additional free movie pass. If $t$ is the number of movie tickets James buys, $d$ is the number of dollars he spends on concessions, and $f$ is the number of free movie passes he receives, the relationship can be expressed by the equation $f = \frac{1}{10}t + \frac{1}{50}d$. Which is the dependent variable in this relationship?
1 Hector types at a rate of 40 words per minute. The equation \( w = 40m \) describes the relationship between \( w \), the number of words he types, and \( m \), the number of minutes he spends typing. Which statement is true?

A The rate at which Hector types depends on the number of minutes he spends typing.
B The number of minutes he spends typing depends on the number of words he types.
C The number of words he types depends on the number of minutes he spends typing.
D The rate at which Hector types depends on the number of words he types.

2 Blaine earns $6 each hour he works at a taco stand. If \( h \) is the number of hours he works, and \( p \) is the total amount of money he makes, what is true about the relationship between \( h \) and \( p \)?

F \( h \) depends on \( p \).
G \( h \) is an independent variable.
H \( p \) is an independent variable.
J Blaine’s hourly rate depends on \( p \).

3 At her job, Mirta can wash 8 cars each hour. The relationship between the number of cars she washes, \( w \), and the number of hours spent washing, \( h \), is described by the function \( w = 8h \). Which statement is true?

A The amount of hours Mirta works depends on the number of cars she washes.
B The amount of money Mirta earns depends on the number of cars she washes.
C The number of cars Mirta washes depends on the number of hours she works.
D The number of cars Mirta washes per hour depends on the number of hours she works.

4 An overnight shipping company charges a flat fee of $10.00, plus $0.50 per ounce the package weighs, and an additional $0.10 per mile of shipment. The price of a shipment can be expressed by the function \( P = 10 + (0.5)w + (0.1)d \), where \( w \) is the weight in ounces, \( d \) is the distance in miles, and \( P \) is the price. Which statement is true?

F The flat fee depends on the total price.
G \( P \) is an independent quantity.
H The distance depends on the weight.
J \( w \) is an independent quantity.

5 Agam pays a $10 membership fee to a gym each month, plus $2 per visitor he brings with him to the gym. If \( c \) is the total amount Agam spends at the gym, and \( v \) is the number of visitors he brings, the relationship between the two variables is \( c = 10 + 2v \). Which statement is true?

A The monthly fee is the only independent variable.
B The number of visitors depends on the total cost.
C The cost is the dependent variable.
D The number of visitors depends on the monthly fee.